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Dimensions of metric graphs and associated parameters

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Abstract

A new outcome on metric dimension is refined herein to obtain: "A diagram G with $\beta(G) = k$ can't have K2k+1 - (2k-1 - 1)e as a subgraph.

Keywords: *Metric Graph, dimensions, Parameters.*

Introduction

As of late in 1996; Samir Khuller, Balaji Raghavachari and Azriel Rosenfeld1 have discussed various parts of the metric dimension of a chart introduced by Harary and have obtained various outcomes. Be that as it may, at times the outcomes have not been amplified to draw a nearer view of those diagrams with little metric dimension. In this paper, we include a few outcomes on charts with metric dimension k which give a glimpse into the milestones managed by them in an extremely simple and rich way. We obtain an improvement of the main outcome Hypothesis 3.2, page 2231 proving that a chart with metric dimension k can't have K2 k+1 − (2k−1 − 1)e as a subgraph. All through this article G indicates a finite simple associated undirected chart. We review that the metric dimension of G, indicated by $\beta(G)$ is defined as the cardinality of a minimal subset S of V having the property that for each pair of vertices u, v in V , there is a w in S to such an extent that d(w, u) 6= d(w, v). The coordinate of every v of V (G) as for every milestone bi belonging to S is defined as expected with i th part as d(v, bi), for every i. The main outcome is obtained via a succession of rudimentary outcomes

Table 1: The code $c_W(v)$ of v with respect to $W = \{u_1, u_7, v_5\}$ in $P(9,3)$.

v	$c_W(v)$	v	$c_W(v)$	v	$c_W(v)$	v	$c_W(v)$	v	$c_W(v)$
u_1	$(0,3,3)$	u_2	$(1,4,2)$	u_3	$(2,4,3)$	u_4	$(3,3,2)$	u_5	$(4,2,1)$
u_6	$(4,1,2)$	u_7	$(3,0,3)$	u_8	$(2,1,2)$	u_9	$(1,2,3)$	v_1	$(1,2,4)$
v_2	$(2,3,1)$	v_3	$(3,3,4)$	v_4	$(2,2,3)$	v_5	$(3,3,0)$	v_6	$(3,2,3)$
v_7	$(2,1,4)$	v_8	$(3,2,1)$	v_9	$(2,3,4)$				

Main Results

Lemma 7. *Let G be a connected graph and let* $|N_2(v)| \geq 6$ *or* $|N_3(v)| \geq 8$ *for each* $v \in V(G)$. *Then* $\dim(G) \geq 3$.

Proof. Clearly, for any $w, v \in V(G)$ and for any $z \in N_k(w)$ we have

$$d(v,w) - k \leq d(v,z) \leq d(v,w) + k. \qquad (7)$$

Suppose, to the contrary, that $S = \{w_1, w_2\}$ is a resolving set of G. Since $|N_2(w_1)| \geq 6$ or $|N_3(w_1)| \geq 8$, we deduce from (7) and the Pigeonhole principle that there exist two vertices $x_1, x_2 \in N_2(w_1)$ such that $d(x_1, w_2) = d(x_2, w_2)$, a contradiction. □

Theorem 8. *For* $n \geq 7$,

(i) If $n \in \{9, 10, 11, 15\}$ *or* $n \equiv 1 \pmod 6$, *then* $\dim(P(n,3)) = 3$.

(ii) If $n = 20$, *then* $\dim(P(n,3)) = 5$.

Proof. If $n = 9$, then let $W = \{u_1, u_7, v_5\}$. The code of v with respect to W in $P(9,3)$ is presented in Table 1 yielding $\dim(P(9,3)) \leq 3$.

Now, we show that $\dim(P(9,3)) \geq 3$. Suppose, to the contrary, there exists a resolving set $W = \{x, y\}$ of $P(9,3)$. First let $W \cap U \neq \emptyset$. We may assume w.l.o.g. that $x \in W \cap U$. By (1), we have $|N_2(x)| = 6$. For each $u \in N_2(x)$, we have $d(y,x) - 2 \leq d(y,u) \leq d(y,x) + 2$. By the Pigeonhole principle, we have $d(y,u) = d(y,v)$ for some $u, v \in N_2(x)$ and this leads to a contradiction. Now let $W \cap U = \emptyset$. Assume without loss of generality that $x = v_1$ and $y = v_i$ for some $i \in \{2,3,4,5\}$. If $i \in \{2,3\}$, then $(d(x,u_4), d(y,u_4)) = (d(x,u_9), d(y,u_9))$, and if $i \in \{4,5\}$, then $(d(x,u_{7-i}), d(y,u_{7-i})) = (d(x,u_{9-i}), (y,u_{9-i}))$, a contradiction. Thus, $\dim(P(9,3)) \geq 3$ and so $\dim(P(9,3)) = 3$.

If $n = 10$, then let $W = \{u_1, v_9, v_{10}\}$. The code of v with respect to W in $P(10, 3)$ is presented in Table 2 showing that $\dim(P(10, 3)) \leq 3$.

Next, we show that $\dim(P(10, 3)) \geq 3$. Suppose, to the contrary, there exists a resolving set $W = \{x, y\}$ of $P(10, 3)$. As above, we may assume that $W \cap U = \emptyset$. We may assume w.l.o.g. that $x = v_1$ and $y = v_i$ for some $i \in \{2, 3, \ldots, 6\}$. If $i \in \{2, 4, 6\}$, then $(d(x, u_3), d(y, u_3)) = (d(x, u_5), d(y, u_5))$, and if $i \in \{3, 5\}$, then we have $(d(x, u_2), d(y, u_2)) = (d(x, u_4), d(y, u_4))$, a contradiction.

If $n = 11$, then let $W = \{u_1, u_5, v_2\}$. The code of v with respect to W in $P(11, 3)$ is presented in Table 3 yielding $\dim(P(11, 3)) \leq 3$.

Table 2: The code $c_W(v)$ of v with respect to $W = \{u_1, v_9, v_{10}\}$ in $P(10, 3)$.

v	$c_W(v)$	v	$c_W(v)$	v	$c_W(v)$	v	$c_W(v)$	v	$c_W(v)$
u_1	(0, 3, 2)	u_2	(1, 2, 3)	u_3	(2, 3, 2)	u_4	(3, 4, 3)	u_5	(4, 3, 4)
u_6	(5, 2, 3)	u_7	(4, 3, 2)	u_8	(3, 2, 3)	u_9	(2, 1, 2)	u_{10}	(1, 2, 1)
v_1	(1, 4, 3)	v_2	(2, 1, 4)	v_3	(3, 2, 1)	v_4	(2, 5, 2)	v_5	(3, 2, 5)
v_6	(4, 1, 2)	v_7	(3, 4, 1)	v_8	(2, 3, 4)	v_9	(3, 0, 3)	v_{10}	(2, 3, 0)

Table 3: The code $c_W(v)$ of v with respect to $W = \{u_1, u_5, v_2\}$ in $P(11, 3)$.

v	$c_W(v)$	v	$c_W(v)$	v	$c_W(v)$	v	$c_W(v)$	v	$c_W(v)$
u_1	(0, 4, 2)	u_2	(1, 3, 1)	u_3	(2, 2, 2)	u_4	(3, 1, 3)	u_5	(4, 0, 2)
u_6	(4, 1, 3)	u_7	(4, 2, 3)	u_8	(4, 3, 3)	u_9	(3, 4, 3)	u_{10}	(2, 4, 2)
u_{11}	(1, 4, 3)	v_1	(1, 3, 3)	v_2	(2, 2, 0)	v_3	(3, 3, 3)	v_4	(2, 2, 3)
v_5	(3, 1, 1)	v_6	(3, 2, 4)	v_7	(3, 3, 2)	v_8	(3, 2, 2)	v_9	(2, 3, 4)
v_{10}	(3, 3, 1)	v_{11}	(2, 3, 3)						

Table 4: The code $c_W(v)$ of v with respect to $W = \{u_1, u_3, v_{11}\}$ in $P(15, 3)$.

v	$c_W(v)$	v	$c_W(v)$	v	$c_W(v)$	v	$c_W(v)$	v	$c_W(v)$
u_1	(0, 2, 4)	u_2	(1, 1, 3)	u_3	(2, 0, 4)	u_4	(3, 1, 4)	u_5	(4, 2, 3)
u_6	(5, 3, 4)	u_7	(4, 4, 3)	u_8	(5, 5, 2)	u_9	(5, 4, 3)	u_{10}	(4, 5, 2)
u_{11}	(5, 5, 1)	u_{12}	(4, 4, 2)	u_{13}	(3, 5, 3)	u_{14}	(2, 4, 2)	u_{15}	(1, 3, 3)
v_1	(1, 3, 5)	v_2	(2, 2, 2)	v_3	(3, 1, 5)	v_4	(2, 2, 5)	v_5	(3, 3, 2)
v_6	(4, 2, 5)	v_7	(3, 3, 4)	v_8	(4, 4, 1)	v_9	(4, 3, 4)	v_{10}	(3, 4, 3)
v_{11}	(4, 4, 0)	v_{12}	(3, 3, 3)	v_{13}	(2, 4, 4)	v_{14}	(3, 3, 1)	v_{15}	(2, 2, 4)

TABLE 5: The code $c_W(v)$ of v with respect to $W = \{u_1, u_3, u_{11}, v_8, v_{10}\}$ in $P(20, 3)$.

v	$c_W(v)$	v	$c_W(v)$	v	$c_W(v)$	v	$c_W(v)$
u_1	(0, 2, 6, 4, 4)	u_2	(1, 1, 5, 3, 5)	u_3	(2, 0, 6, 4, 4)	u_4	(3, 1, 5, 3, 3)
u_5	(4, 2, 4, 2, 4)	u_6	(5, 3, 5, 3, 3)	u_7	(4, 4, 4, 2, 2)	u_8	(5, 5, 3, 1, 3)
u_9	(6, 4, 2, 2, 2)	u_{10}	(5, 5, 1, 3, 1)	u_{11}	(6, 6, 0, 2, 2)	u_{12}	(5, 5, 1, 3, 3)
u_{13}	(6, 6, 2, 4, 2)	u_{14}	(5, 5, 3, 3, 3)	u_{15}	(4, 6, 4, 4, 4)	u_{16}	(5, 5, 5, 5, 3)
u_{17}	(4, 4, 4, 4, 4)	u_{18}	(3, 5, 5, 5, 5)	u_{19}	(2, 4, 6, 4, 4)	u_{20}	(1, 3, 5, 5, 5)
v_1	(1, 3, 5, 5, 3)	v_2	(2, 2, 4, 2, 4)	v_3	(3, 1, 5, 5, 5)	v_4	(2, 2, 4, 4, 2)
v_5	(3, 3, 3, 1, 5)	v_6	(4, 2, 4, 4, 4)	v_7	(3, 3, 3, 3, 1)	v_8	(4, 4, 2, 0, 4)
v_9	(5, 3, 3, 3, 3)	v_{10}	(4, 4, 2, 4, 0)	v_{11}	(5, 5, 1, 1, 3)	v_{12}	(4, 4, 2, 4, 4)
v_{13}	(5, 5, 3, 5, 1)	v_{14}	(4, 4, 2, 2, 4)	v_{15}	(3, 5, 3, 5, 5)	v_{16}	(4, 4, 4, 4, 2)
v_{17}	(3, 3, 3, 3, 5)	v_{18}	(2, 4, 4, 6, 4)	v_{19}	(3, 3, 5, 3, 3)	v_{20}	(2, 2, 4, 4, 6)

Conclusion

At present, we are approaching a few issues connected with the metric dimension of diagrams. Some of them are the following:

- Obtaining a superior upper headed for the metric dimension of the cartesian result of two charts. To be more precise, we are trying to demonstrate (or finding a counterexample) that for all pair of charts G, H: $\beta(GH) \leq \beta(G)+\beta(H)$.
- Computing the metric dimension in the class of hypercubes (a few old and new realized values are displayed in the table underneath), grid charts and Hamming diagrams.

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